AMYGDALA $z \mapsto z^2 + c$

A Newsletter of fractals & $\mathcal{M}$ -- the Mandelbrot Set
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Issue #5
July 1987

CONTENTS

A MANDELBROT MACHINE

From Ulrich Schmidt (June 27, 1987):

Dear Rollo,

Your surface mail arrived here on the 20th June, it was post-stamped 28th April. This delay of almost two months is excessive. If you need to charge more for air-mail delivery, please do so by all means — I am quite willing to pay for this service. Surface mail, however, is not tolerable *at any rate*.

Since I have taken pen to paper (metaphorically speaking) I might as well tell you about my latest Mandelbrot computations. I have just finished construction of a Mandelbrot machine. It consists of two Weitek floating-point chips (an adder and a multiplier), two register files which can be configured as 64 32-bit or 32 64-bit registers, and microcoded control logic built of bit-slice components (2910, 29101).

The microcode resides in fast static RAM; it can be downloaded from the host to try out different algorithms. The word width is 128 bits to support the bandwidth of the Weitek chips. The Mandelbrot machine communicates with the outside world via a 16K dual-port buffer, accepting parameters and depositing results. The whole thing fits on a double-size Eurocard and plugs into a VMEbus slot.

How fast is it? Well, the adder and multiplier chips deliver a result every 125 ns (250 ns for the multiplier when using double precision), thus yielding an upper bound of 16 MegaFLOP per second. This result can only be achieved if the pipelines of the floating-point chips are kept full all of the time. In general, this is not possible with Mandelbrot computations because of the iterative nature of the algorithm. However, by computing several points *quasi-simultaneously*, i.e. by interleaving several independent computations, we can approach the theoretical limit very closely.

I have tried out several interleaving schemes for single as well as for double precision Mandelbrot computations. In single precision mode, I compute six neighboring points simultaneously versus three points in double precision mode. There is a certain overhead in this method as no new points are introduced until *all* points in the six-pack have been computed. This overhead is practically negligible because all points belonging to M have to be computed up to the iteration limit anyway, and of course these computations make up the largest part of the running time.

What it all comes down to is this: it takes 8 seconds to compute the base picture with coordinates (-2.25, 0.75, -1.5, 1.5), a display resolution of 512 x 512 and an iteration limit of 256 when using single precision (14 seconds for double precision). These times include the overhead to display the points on a colour monitor. I challenge your readers to beat this time, by using whatever computing resource (Cray?) they can get hold of. (A system with five transputers took 40 seconds).

By the way, I have also put a 68020/68881 board (16 MHz, no wait states) into my VMEbus system. This board was used in parallel to the vector processor to "help" with the Mandelbrot computations, but its use didn't make any difference — it was 30 - 50 times slower than the Mandelbrot vector processor. When was the last time you heard a 68020/68881 combo called *slow?*

Best regards,

Ulrich Schmidt
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5100 Aachen, West Germany

(Continued on page 2)

RS replies:

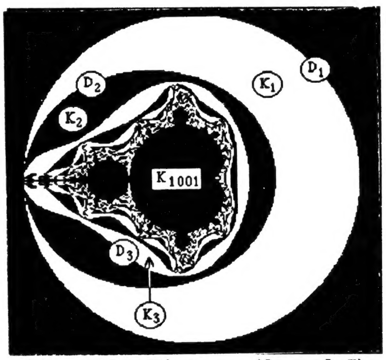
Unfortunately, I sent out Amy #4 *surface rate* to all foreign subscribers except the ones in Australia and New Zealand. I had no idea at that time that the delay would be so long. I will send out Amy #5 and future issues as Printed Matter, Air Mail, at no extra cost.

Your Mandelbrot machine sounds fascinating; I'd like to hear more about it. For now, I have a few questions. What clock speed do you operate at? (8 MHz?) What do you use for a host machine? How did you construct the board: wire wrap? Multiwire? Do you use a microcode compiler or assembler to translate the microcode? Are you planning to develop the Mandelbrot machine as a product? If so, at what cost? Would you be willing to sell or share its design, e.g. provide schematics, layout, design notes, microcode logic?

A JOURNEY TO THE WEST

by Esseh N. Namreh

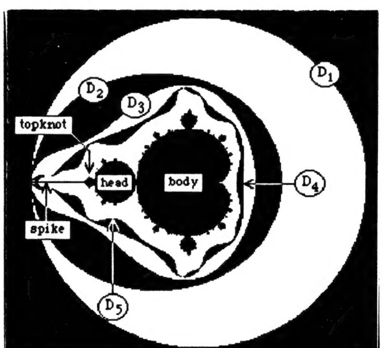
We're off on a tour of the West, with the program EMD as our map, compass, and guide. All views shown have pixelation 251x251, and have dwell limit 1000. We start with a map of the only interesting region in the entire universe: the disk of radius two centered on the origin of the complex plane (center 0, magnification 1/2). Here it is: the Big Picture: →



The Big Picture (0 x 0.5)

The circle of radius 2 centered on zero is the boundary D_1 of the large white disk (see TUTORIAL: MANDELBROT POLYNOMIALS). The inside of D_1 is K_1, which contains all points of dwell 1 or greater. The black region outside that circle extends to infinity in all directions and contains all points having dwell zero.

Moving inward, we come to the boundary D_2 of the elliptical black region K_2, centered on -.5. It contains all points having dwell 2 or greater. Continuing inward, we come to the boundary D_3 of the pear-shaped white region K_3, then to D_4, D_5, etc. Because of the graininess of the pixelation, these successive contours cannot be seen as they really are, but break up into a visual mess. We can suppress the mess using a feature of EMD: by typing *t4* we color *white* all points between D_5 and the contour D_{1000} which is associated with the dwell limit, while coloring K_{1000} itself *black*.

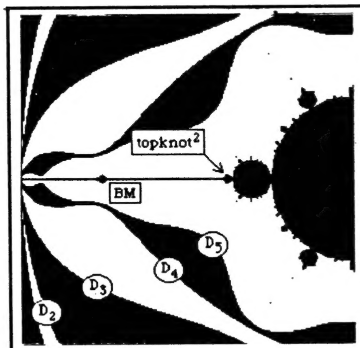


The Big Picture Enhanced (0 x 0.5)

What appears to be the Mandelbrot set is, strictly speaking, the region K_{1000}: the set of all z for which $|M_{1000}(z)| \leq 2$.

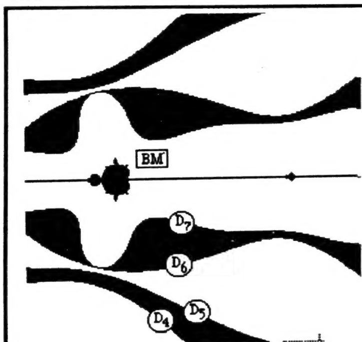
The goal of our journey is the point -2, the "Utter West", which is the tip of the spike extending in the negative real direction from $\mathcal{M}$. As we travel toward the Utter West we will examine various exotic, beautiful and astonishing features, increasing magnification as necessary. Before moving on let's note four large-scale features of $\mathcal{M}$: the *body, head, topknot, and spike,* as labeled in the diagram. These will help orient us as we move about and change magnification.

In the next figure, *Westward Ho!*, we have increased magnification by a factor of 4 (from 1/2 to 2) and moved the center of our view to -1.5. The increased resolution reveals that the topknot itself has a topknot, which, I suppose, is *topknot*^{2} (and presumably *topknot*^{2} has a *topknot*^{3}, etc.), and that skewered upon the spike is a blob, called *BM*, for reasons that will become clear shortly. The contours D_2-D_5 are labeled to provide continuity with the previous diagram.



Westward, Ho! (-1.5 x 2)

There are lots of interesting features between topknot^2 and BM (an infinite number, in fact!), but we won't stop to investigate. Let's move on to a view which centers on BM (at –1.7), and increase our magnification by another factor of 4, to 8. We also increase the threshold to reveal two additional contours, D_6 and D_7. $\mathcal{M}$ is surrounded by an infinity of miniature $\mathcal{M}$ s, an infinity of which are skewered by the spike. We call these *midgets*, and the biggest of these is *BM*, the Biggest Midget. It is followed at a discreet distance by a mascot which, if we stopped to investigate, would no doubt turn out to be another midget.



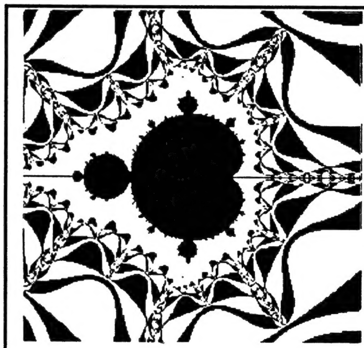
The Biggest Midget (-1.7 x 8)

You should have noticed by now that the contours are wavy, and that the number of waves seems to double as we move from one contour to the next inner one.

Now let's get a closer look at BM by centering on it and increasing our magnification to 32. At the same time we in-

crease the threshold to 18 to reveal more of the surrounding contours.

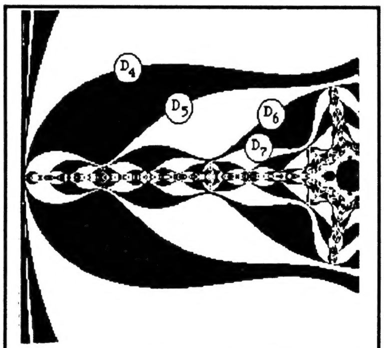
Looking back over the previous figures, you can spot a curious pattern: where the contours press in toward the real axis, the compression seems to create a Mandelbrot replica, much as heat and pressure create a diamond. The compression of D_2 creates the body; that of D_3 the head; that of D_5 the topknot.



The Biggest Midget (-1.757 x 32)

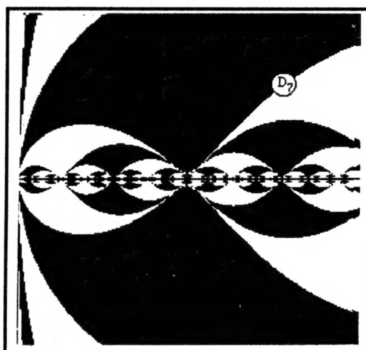
This phenomenon has to do with the field lines of the electric field set up by $\mathcal{M}$ as an electrically charged body. Each such field line extends to infinity, ultimately straightening out to become asymptotic to a ray arg(z) = const. If the angle arg(z) is a rational multiple of π the field line originates on the boundary $\partial\mathcal{M}$ of $\mathcal{M}$ (see *TBF* [1] for details). Much as the bubble trail in a bubble chamber betrays the presence of an invisible charged particle, a "pinch" betrays the presence of a field line originating on $\partial\mathcal{M}$. By a symmetry argument, the originating point cannot be on the Spike, hence there is a local bulge.

Let's now return to a view which spans the range from BM on the East to the Uttermost West. When rotated 90 degrees clockwise, this is essentially the same view as the "Darth Vader" color picture that was sent to some of you a while ago.

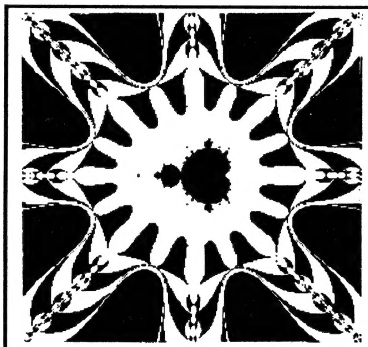


From the Biggest Midget to the Uttermost West (-1.878 x 8)

Over on the left-hand side of this picture is the first pinch of D_7, which creates the John Dewey Jones midget at –1.99638. We examine that midget first with magnification 276, which allows the picture to include the Utter West. Next we zoom in with a magnification of 70,000, suppressing contours beyond D_{24}. This produces a black and white rendition of the view depicted in the color slide that was distributed with Amygdala #2. →



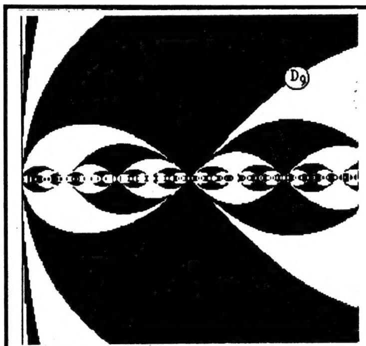
The John Dewey Jones Midget (-1.99638 x 276)



John Dewey Jones (-1.99638 x 70,000)

We've encountered several interesting midgets on our way west. What wonders await us within the singularity at -2, the Utter West?

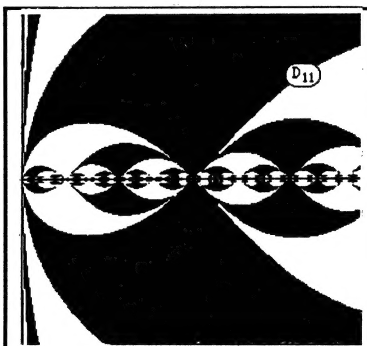
We can glimpse it at the extreme left of the previous figure: (-1.99638 x 276), a figure consisting of nested infinity-signs; but what will increasing magnification reveal? The result of magnification 4400 is shown at the right. Nothing much yet!



The Uttermost West (-1.99977 x 4400)

Let's try magnification 70,000, as shown in the next picture below.

What's going on here? Is the singularity laughing at us, hiding its secrets from our intense probing? But there are no secrets! Increased magnification only expresses the "smooth fractal" nature of the singularity, which consists of nothing but endless Yin/Yang loops spinning endlessly inward and eastward — infinity nested within infinity — a delicious irony!



The Uttermost West (-1.999986 x 70,000)

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[1] H.-O. Peitgen and P.H. Richter, <u>The Beauty of Fractals</u>, Chapter 5, <u>External Angles and Hubbard Trees</u>. Springer-Verlag, Berlin/New York 1986.

TUTORIAL: MANDELBROT POLYNOMIALS

Another view of the Mandelbrot set and Julia sets for the quadratic map $z \mapsto z^2+c$ can be gained by considering the Mandelbrot polynomials:

$M_0(z) = 0$

$M_1(z) = z$

$M_2(z) = z^2 + z$

$M_3(z) = z^4 + 2z^3 + z^2 + z$

and in general:

$M_{n+1}(z) = [M_n(z)]^2 + z,\ n = 0,1,...$

M_n is obviously a polynomial of degree 2^{n-1} with integer coefficients.

Let's also define the critical disk $K = \{z: |z| \leq 2\}$[2], and its boundary $D = \{z: |z| = 2\}$.

We will also need the inverse images of K and D under the mappings M_n:

$K_n = \{z: M_n(z) \in K\}$

and

$D_n = \{z: M_n(z) \in D\}$

The **dwell** of a complex number z is the largest n for which $z \in K_n$, i.e. $|M_n(z)| \leq 2$. If all $|M_n(z)| \leq 2$, the dwell of z is infinite.

The nice thing about the Mandelbrot polynomials is that they are nice and smooth. It is easier to handle them than it is to handle dwells directly.

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[2] The set of all z such that $|z| \leq 2$).

DOES THE MANDELBROT SET CONTAIN INTELLIGENT LIFE?

Our immediate response to the question is

"Don't be absurd! What a ridiculous idea!" So powerful is this response that we must examine its basis before we can proceed with our inquiry.

Challenged to define *intelligence,* I'd probably come up with something like

"Ability to respond appropriately to novel situations."

Note how *time* is an essential part of this definition: only in a *novel* situation can we distinguish an intelligent response from mere instinct; only if the object of our study *responds* do we have anything to evaluate. Nothing within the Mandelbrot Set changes over time. Therefore it is absurd to discuss its intelligence. Subject closed.

But perhaps this is a little too hasty. After all, our proof also demonstrates that God, as conceived of by the world's major religions, cannot possibly be intelligent. While some of us might be prepared to accept that conclusion, we really cannot claim it to be self-evident. Let us therefore re-examine the reasoning that led to it.

We could attempt to strip away the element of time from our definition of intelligence, or we could attempt to view the object of our study as changing in time. The latter course, I think, comes more naturally to us. Consider a cellular automaton like Conway's Life. We imagine watching the cells live and die... gliders traverse the screen... after a long time self-replicating patterns of cells evolve... they compete for space on the plane... it does not seem too difficult to imagine seeing phenomena here that we would call intelligent. But all we have seen changing on a two-dimensional plane we could also describe as a static three-dimensional pattern: we can imagine the patterns of successive generations etched on glass plates and stacked one above the other, the histories of individual groups of cells visible as branching clusters of thread within the stack. The mathematics that generate Conway's Life do not distinguish between these two descriptions. Yet one way of looking allows us to see the possibility of intelligence within the Life World, the other rules out that possibility as absurd.

It seems, then, that we suffer from an unfortunate temporal chauvinism. To recognize intelligence within a system, we must transform our view of that system so that it appears to change with time. We accept that this transformation may not alter the mathematical nature of the system, that it is merely a concession to our inability to characterize a timeless intelligence. That granted, what can we see when we look at the Mandelbrot Set?

There are many ways of introducing a temporal element into our description of the Set. One of the simplest is to identify the upper limit of dwell time with elapsed time. At the first second, we approximate the Set as all those points whose magnitude remains less than two after a single iteration: a featureless circular blob. As the seconds pass, the boundaries of the blob shrink and bud; differentiated structures appear. We see analogies with the growth of a fœtus, or the progressive folding and wrinkling of the cerebral cortex in the evolution of the higher primates. The unfolding structures grow finer and more intricate; at length we can no longer take in the whole picture, and must concentrate on fragments. Eventually we reach the limit of our computer's resolving power. But this is a limit on our own vision, not on the Set itself. The Set's fine structure continues to evolve beyond the limit of our vision, becoming ever more complex... and this process must be imagined as going on *eternally,* far outlasting the puny span of geological time. Can we not conceive of intelligence arising somewhere within these convoluted depths?

But, the reader objects, all this structure, however complex and extensive, is fully determined by a simple mathematical formula. Intelligence implies the possibility of choice between alternatives, but the Set's evolution is fixed in advance. It is logically impossible for it to deviate in any way from the pattern determined by the iron laws of mathematics. To imagine otherwise is mere fantasy.

This objection does not deserve much respect. It might equally well be argued that *we* cannot be intelligent, since we are governed by the iron laws of physics. If we have not yet resolved the Free-Will-versus-determinism question in our own case, we cannot reasonably turn to its arguments for enlightenment in the case of the Mandelbrot Set.

I hope to have established that the possibility of intelligent life within the Set is at least worthy of consideration. Of course, once we have opened this question up for discussion, a host of related questions occur to us: are there many intelligences within the Set or only one? (The second alternative might, by analogy with the Gaia Hypothesis, be termed the Amygdala Hypothesis.) If there is intelligent life within the

Set, what is it thinking about? And finally — though this seems just an impossible dream — if there were intelligent life within the Set, could we ever make contact with it? (Readers who believe they have been contacted by Amygdalartoidal intelligences are urged to write in describing their experiences.)
John Dewey Jones

FRACTAL SOFTWARE REVIEWS

In Amy #2 I proposed that there be reviewers and distributors of software for various computers. I took on the job for the Macintosh, and asked for volunteers from the subscribers to take care of other computers.

Thomas Granvold volunteers:

... to be contact for Commodore Amiga computers, "Contact Amiga". I currently have two disks of fractal software and pictures. The contents of these are in the public domain. Interested people can get copies of these from me, at the address below, for a $6.00 fee per disk. That seems to be the going rate for a disk of public domain Amiga software.

Amiga FSD #1 contains three Mandelbrot image generating programs; it is great to have 880K bytes per disk. The programs are:
Mandelbrot Explorer Set by T. Wilcox [also does 3D plots].
Mand by R. French and R.J. Mical [including the source code in C].
Mandelbrot by S.H. Landrum [including the source code in C and assembly].
Amiga FSD #2 contains images of the Mandelbrot set and a public domain "slide show" program that will display the images. These are the most interesting images that I have generated or received from others. They are all in IFF format, which means that they can be used by most Amiga programs that handle graphic images.

Both of these disks have room left on them, so as I find new material I will add to their contents. I am currently tracking down programs that use fractals to generate trees and mountains.

*Address, **Amiga** Contact:*

Tom Granvold
1087 C Reed Ave
Sunnyvale, CA 94086

LETTERS

From Jim Loudon:

Please institute a *Questions* column in *Amygdala.* I can get it started all by myself, including questions I asked you in my 1986 August 28 letter requesting #0, to which you responded by sending #0 but in no other way... [there follow four questions which appear in the **QUESTIONS** column, below]...

Please indicate in some way which direction is supposed to be which on your slides. I know from VERY MUCH EXPERIENCE, e.g. lecturing from NASA pictures of Earth from space, that reproduced versions of slides are often given me with no indication of which way is north, let alone which side of the slide should face the screen.

If you accept T-shirt notices (e.g. Amygdala #3, page 6), PLEASE require them to state WHAT THE SHIRTS ARE MADE OF, for the huge number of us out here who are allergic to polyester and need 100% COTTON! Ditto for bumper stickers: vinyl lasts; paper (cheaper) rapidly decays in place.

RS replies:

The slides distributed with Amy #2 and 3 are embossed with the legend "THIS SIDE TOWARD SCREEN". Both are bilaterally symmetric. If #37 (the mandala with the miniature $\mathcal{M}$ in the middle) is oriented so that the bean-shaped body of the set faces right and the head faces left, the real axis is horizontal, with values increasing to the right, which is the conventional orientation. #35 should be oriented with the large black area to the left and the colored extrusion to the right.

QUESTIONS

From Jim Loudon:

JL: Is $\mathcal{M}$ really a fractal? My understanding of the term "fractal" includes that it looks like itself as you go indefinitely to smaller and smaller sizes. I also understand that $\mathcal{M}$ doesn't do that; it gets more and more complex as you go to smaller and smaller sizes.

RS: It is a mistake to attempt to define the term "fractal" narrowly. Professor Mandelbrot introduces it in *The Fractal Geometry of Nature* as follows (page 1):

Why is geometry often described as "cold" and "dry"? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.

More generally, I claim that many patterns of Nature are so irregular and fragmented, that, compared with *Euclid* — a term used in this work to denote all of standard geometry — Nature exhibits not simply a higher degree but an altogether different level of complexity. The number of distinct scales of length of natural patterns is for all practical purposes infinite.

The existence of these patterns challenges us to study those forms that Euclid leaves aside as being "formless," to investigate the morphology of the "amorphous." Mathematicians have disdained this challenge, however, and have increasingly chosen to flee from nature by devising theories unrelated to anything we can see or feel.

Responding to this challenge, I conceived and developed a new geometry of nature and implemented its use in a number of diverse fields. It describes many of the irregular and fragmented patterns around us, and leads to full-fledged theories, by identifying a family of shapes I call *fractals.* The most useful fractals involve *chance* and both their regularities and their irregularities are statistical. Also, the shapes described here tend to be *scaling,* implying that the degree of their irregularity and/or fragmentation is identical at all scales. The concept of *fractal* (Hausdorff) *dimension* plays a central role

in this work.

Some fractal sets are curves or surfaces, others are disconnected "dusts," and yet others are so oddly shaped that there are no good terms for them in either the sciences or the arts. The reader is urged to sample them now, by browsing through the book's illustrations.

RS: You should get a copy of *The Fractal Geometry of Nature,* Jim. The sections on the "Fractal View of Galaxy Clusters" and "Conditional Stationarity and Cosmographic Principles" will expand your astronomer's consciousness, to say the least.

JL: What's so magic about the function $z^2 + c$ that it has a special name, the "Mandelbrot Set"? Is there something about it that makes it richer than other iterative functions one might think of, e.g. (to take a simple, obvious example) $z^3 + c$?

RS: The function $f(z) = z^2 + c$ is perhaps the simplest function of a complex variable whose iteration is interesting. The Mandelbrot set is not the function itself, but the set of all c for which the iterates remain bounded. Its name honors Professor Mandelbrot, who discovered it. It may or may not be "richer" than $f(z) = z^3 + c$, whose iteration is also interesting, but it was studied first (See FGN, page 188 ff.).

If you study *The Beauty of Fractals,* you will find that the image of $\mathcal{M}$ occurs in many contexts, some far removed from the iteration of $f(z) = z^2 + c$, and for deeply significant reasons (see e.g., TBF, section 9, on Yang-Lee Zeros).

JL: If $\mathcal{M}$ is a fractal, how many dimensions does it have. It's connected, so its boundary has a specific length: is that infinite, finite (if so, how long?) or unknown as yet?

RS: The boundary of $\mathcal{M}$ is a fractal with dimension somewhere between 1 and 2. It is easy for me to answer the question of what its exact dimension is: I dunno. Maybe John Hubbard does. I'm sure it's of infinite length.

JL: How should one pronounce "Mandelbrot", i.e. how does he pronounce it?

RS: I'll defer the answer to this question to Professor Mandelbrot, if he cares to answer it. I think I've bandied his name about enough in these pages.

BIBLIOGRAPHY

The official Amygdala bibliographer is:

Thomas Bank
933 Portland Place, Apt #16
Boulder, CO 80302
(303) 443-1809

He will:

° Maintain a bibliography of articles, books, videos and movies on the subjects of fractals, chaos, the Mandelbrot set, and Julia sets. Ideally, each entry should include a short note summarizing its content.

° Offer the bibliography for a nominal cost (copying and postage) to all, not just Amygdala subscribers.

Readers of Amygdala should submit article and book bibliographic entries directly to him to be added to the list.

52. (Cover picture) *Science News* 131,8 (May 2, 1987). "The black spleenwort fern shown here, as seen from several different angles, wasn't grown in a flower pot or in a shady spot by a stream, but on a computer. This set of fractal images was computed from a simple set of equations, which captured the fine details evident in a real fern. A similar method can be used for image compression, permitting a lot of information to be conveyed by a small amount of data. (Illustration: Barnsley © 1987, Georgia Tech Research Group)" [See also citation #53]

53. I Peterson, "Packing it In",. *Science News* 131,8 (May 2, 1987) 283-285. [Discusses the work of Michael F. Barnsley and colleagues at Georgia Tech on fractal encoding and image compression. The idea is to take an image — of a leaf, say — and find smaller, compressed, rotated, translated, possibly stretched copies of it which when fitted together and piled up so that they partially overlap form a "collage" which approximates the original. The original images can then be discarded, retaining the "affine" maps which specify the transformations producing the copies. The original image can be approximately reconstituted from a single point by iteratively applying the maps to it. A description of the maps themselves constitutes a highly compressed encoding of the original image. See also citation #52.]

FOREIGN COLLECTIONS

Subscribers outside the US need to pay us in funds drawn on a US bank, or include an extra $10.00 for foreign collection — trying to collect foreign funds from a US bank is *expensive!*

ERRATA

In Issue #4, EXPLORING THE INTERIOR OF $\mathcal{M}$, I defined inf(z) as the minimum value of $|z_n|$, where z0 = 0 and $z_{n+1} = z_n^2 + z$. It is, of course the minimum value of $|z_n|$ **for $n > 0$.**

COMMERCIAL PRODUCTS

If you place an order as a result of seeing the following notices, please mention Amygdala with your order.

ART MATRIX; PO Box 880; Ithaca, NY 14851 USA. (607) 277-0959. Prints, FORTRAN program listings, 36 postcards $7.00, sets of 2 packs $10.00, 140 slides $20.00. Or send for FREE information pack with sample postcard. Custom programming and photography by request. Make a bid.

CIRCULATION

As of June 28, 1987 Amygdala has 149 paid-up subscribers, of which 34 have the supplemental color picture subscription.